# Math HL 12 Summer Review Packet

In order to keep your current math skills sharp, please complete the summer review packet. Please complete before the first day of school.

Show all works and graphs clearly.
Solutions are not provided with this review packet.

Have a good summer! CDS Mathematics Department

# 12 Math HL Summer packet 2021 Name______________

**Due at the first day of class in August 2021, Calculator allowed.**

**1.** *[2 marks]* Consider the triangle $\mathrm{PQR}$ where $\mathrm{Q\hat{P}R} = 30^\circ$, $\mathrm{PQ} = (x+2)\ \mathrm{cm}$ and $\mathrm{PR} = (5-x)^2\ \mathrm{cm}$, where $-2 < x < 5$.

Show that the area, $A\ \mathrm{cm}^2$, of the triangle is given by $A = \frac{1}{4}(x^3 - 8x^2 + 5x + 50)$.

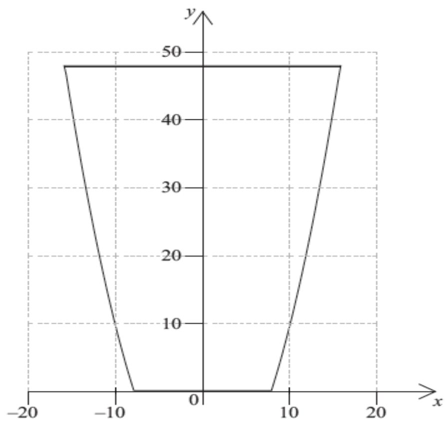
**1b.** *[3 marks]* (i) State $\frac{\mathrm{d}A}{\mathrm{d}x}$. (ii) Verify that $\frac{\mathrm{d}A}{\mathrm{d}x} = 0$ when $x = \frac{1}{3}$.

**1c.** *[7 marks]* (i) Find $\frac{\mathrm{d}^2A}{\mathrm{d}x^2}$ and hence justify that $x = \frac{1}{3}$ gives the maximum area of triangle $PQR$.

(ii) State the maximum area of triangle $PQR$.

(iii) Find $QR$ when the area of triangle $PQR$ is a maximum.

**2a.** *[3 marks]* The vertical cross-section of a container is shown in the following diagram.




The curved sides of the cross-section are given by the equation $y = 0.25x^2 - 16$. The horizontal cross-sections are circular. The depth of the container is $48$ cm.

If the container is filled with water to a depth of $h$ cm, show that the volume, $V$ cm$^3$, of the water is given by $V = 4\pi\left(\frac{h^2}{2} + 16h\right)$.

**2b.** *[10 marks]*

The container, initially full of water, begins leaking from a small hole at a rate given by $\frac{\mathrm{d}V}{\mathrm{d}t} = -\frac{250\sqrt{h}}{\pi(h+16)}$ where $t$ is measured in seconds.

(i) Show that $\frac{\mathrm{d}h}{\mathrm{d}t} = -\frac{250\sqrt{h}}{4\pi^2(h+16)^2}$.

(ii) State $\frac{\mathrm{d}t}{\mathrm{d}h}$ and hence show that $t = \frac{-4\pi^2}{250}\int\left(h^{\frac{3}{2}} + 32h^{\frac{1}{2}} + 256h^{-\frac{1}{2}}\right)\mathrm{d}h$.

(iii) Find, correct to the nearest minute, the time taken for the container to become empty. ($60$ seconds = 1 minute)

**3.** *[4 marks]* One root of the equation $x^2 + ax + b = 0$ is $2 + 3\mathrm{i}$ where $a,\ b \in \mathbb{R}$ Find the value of $a$ and the value of $b$.

**4a.** *[5 marks]* Let $f(x) = x(x+2)^6$.

Solve the inequality $f(x) > x$.

**4b.** *[5 marks]* Find $\int f(x)\mathrm{d}x$.

**5a.** *[5 marks]* Consider the curve with equation $\left(x^2 + y^2\right)^2 = 4xy^2$.

Use implicit differentiation to find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$.

**5b.** *[3 marks]* Find the equation of the normal to the curve at the point (1, 1).

**6a.** *[4 marks]*

The function $f$ is defined as $f(x) = -3 + \frac{1}{x-2},\ x \neq 2$.

(i) Sketch the graph of $y = f(x)$, clearly indicating any asymptotes and axes intercepts.

(ii) Write down the equations of any asymptotes and the coordinates of any axes intercepts.

**6b.** *[4 marks]*

Find the inverse function $f^{-1}$, stating its domain.

**7.** *[5 marks]* Sand is being poured to form a cone of height $h$ cm and base radius $r$ cm. The height remains equal to the base radius at all times. The height of the cone is increasing at a rate of $0.5\ \text{cm min}^{-1}$.

Find the rate at which sand is being poured, in $\text{cm}^3\ \text{min}^{-1}$, when the height is 4 cm.