2018 Summer Review for Students Entering Algebra 1

1. Order of Operations
2. Unit Rates and Proportions
3. Area and Perimeter
3. Solving Equations
4. Multiple Representations of Linear Equations
5. Transformations
6. Scatterplots
7. Box Plots

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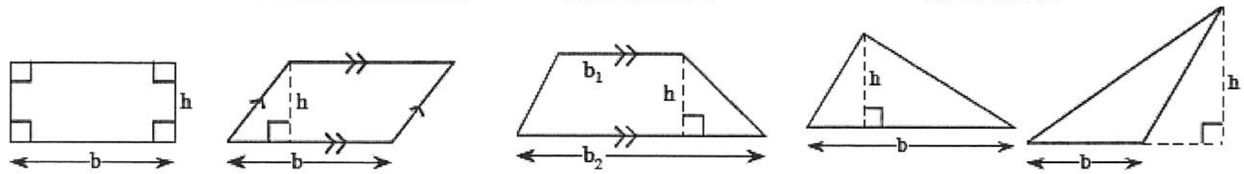
Area and Perimeter of Polygons

Area is the number of square units in a flat region. The formulas to calculate the areas of several kinds of quadrilaterals or triangles are:

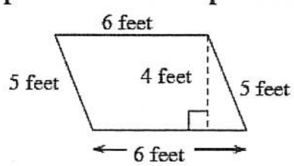
RECTANGLE PARALLELOGRAM TRAPEZOID TRIANGLE



$A = bh$ $\quad$ $A = bh$ $\quad$ $A = \frac{1}{2}(b_1 + b_2)h$ $\quad$ $A = \frac{1}{2}bh$

Perimeter is the number of units needed to surround a region. To calculate the perimeter of a quadrilateral or triangle, add the lengths of the sides.

Example 1:
Compute the area and perimeter.

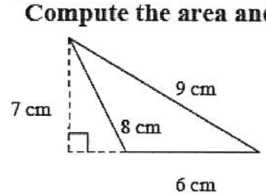


parallelogram

$A = bh = 6 \cdot 4 = 24\ \text{feet}^2$

$P = 6 + 6 + 5 + 5 = 22\ \text{feet}$

Example 2:
Compute the area and perimeter.



triangle

$A = \frac{1}{2}bh = \frac{1}{2} \cdot 6 \cdot 7 = 21\ \text{cm}^2$

$P = 6 + 8 + 9 = 23\ \text{cm}$

Order of Operations

The acronym PEMDAS may help: (Please Excuse My Dear Aunt Sally)

- First evaluate expressions in parentheses
- Evaluate each exponential (for example, $5^2 = 5 \cdot 5 = 25$).
- Multiply and divide each term from left to right.
- Combine like terms by adding and subtracting from left to right.

Numbers above or below a "fraction bar" are considered grouped. A good way to remember is to circle the terms like in the following example. Remember that terms are separated by + and – signs.

Example 1: Simplify $\quad 12 \div 2^2 - 4 + 3(1 + 2)^3$

Simplify within the circled terms: Be sure to perform the exponent operations before dividing.

$12 \div 2^2 = 12 \div 2 \cdot 2 = 3$

Then perform the exponent operation: $3^3 = 3 \cdot 3 \cdot 3 = 27$
Next, multiply and divide left to right: $3(27) = 81$
Finally, add and subtract left to right: $3 - 4 = -1$

$12 \div 2^2 - 4 + 3(1+2)^3$

$3 - 4 + 3(3)^3$

$3 - 4 + 3(27)$

$3 - 4 + 81$

$-1 + 81$

80

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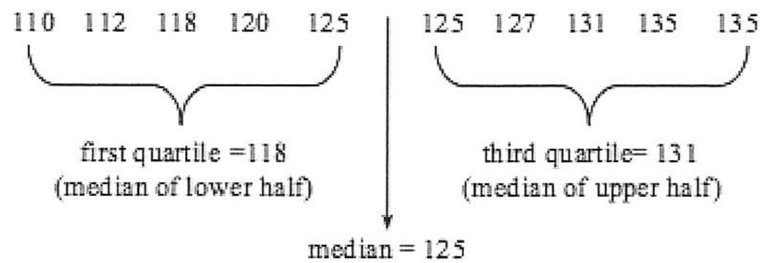
Box Plots

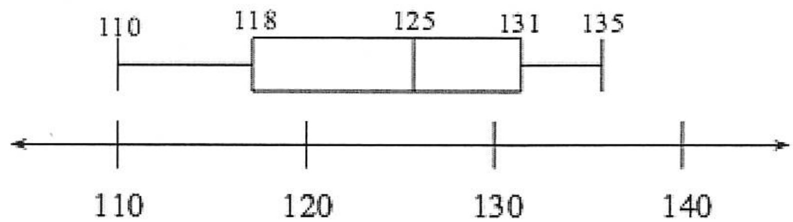
A box plot displays a summary of data using the median, quartiles, and extremes of the data. The box contains the "middle half" of the data. The right segment represents the top 25% of the data and the left segment represent the bottom 25% of the data.



Example: Create a box plot for the set of data given in the previous example.

Solution:

Place the data in order to find the median (middle number) and the quartiles (middle numbers of the upper half and the lower half.)



Based on the extremes, first quartile, third quartile, and median, the box plot is drawn. The interquartile range IQR = 131–118 = 13.

Unit Rates and Proportions

A rate is a ratio comparing two quantities. A unit rate is simplified to have a denominator of 1. Either unit rates or proportions (equations with ratios) can be used to solve ratio problems. Solutions can also be approximated by looking at graphs of lines passing through the origin and the given information.

Example 1: Sam paid \$4.95 for $\frac{3}{4}$ pound of his favorite trail-mix. What is the unit price (dollars per pound)?

Solution: The unit price is $\frac{\$4.95}{\frac{3}{4}\text{ pound}}$. To create a unit rate we need a denominator of "one."

$$\frac{\$4.95\cdot\frac{4}{3}}{\frac{3}{4}\text{ pound}\cdot\frac{4}{3}} = \frac{\$4.95\cdot\frac{4}{3}}{1\text{ pound}} = \$4.95\cdot\tfrac{4}{3} = \$6.60$$

Example 2: Based on the table, what is the unit growth rate (meters per year)?

+2

Height (m)	15	17
Years	75	85

+10

Solution:

$$\frac{2\text{ meters}}{10\text{ years}} = \frac{2\text{ meters}\div 10}{10\text{ years}\div 10} = \frac{\frac{1}{5}\text{ meter}}{1\text{ year}} = \frac{1}{5}\ \frac{\text{meter}}{\text{year}}$$

Example 3: In line at the movies are 146 people in front of you. If you count 9 tickets sold in 70 seconds, how long will it take before you buy your ticket?

Solution: The information may be organized in a proportion $\frac{9\text{ tickets}}{70\text{ seconds}} = \frac{146\text{ tickets}}{x}$. Solving the proportion $\frac{9}{70} = \frac{146}{x}$ yields $9x = 10220$, so $x \approx 1135.56$ seconds or $\approx$ 19 minutes.

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Solving Equations

Equations in one variable may be solved in a variety of ways. Commonly, the first step is to simplify by combining like terms. Next isolate the variable on one side and the constants on the other. Finally, divide to find the value of the variable. Note: When the process of solving an equation ends with different numbers on each side of the equal sign (for example, 2 = 4), there is *no solution* to the problem. When the result is the same expression or number on each side of the equation (for example, $x + 3 = x + 3$) it means that *all real numbers* are solutions.

Example 1: Solve $4x + 4x - 3 = 6x + 9$

Solution:		Check:
$4x+4x-3=6x+9$	problem	$4(6)+4(6)-3=6(6)+9$
$8x-3=6x+9$	simplify	$24+24-3=36+9$
$2x=12$	add 3, subtract $6x$ on each side	$48-3=45$
$x=6$	divide	$45=45$

Example 2: Solve $-4x + 2 - (-x + 1) = -3 + (-x + 5)$

Solution:		Check:
$-4x+2-(-x+1)=-3+(-x+5)$	problem	$-4\left(-\frac{1}{2}\right)+2-\left(-\left(-\frac{1}{2}\right)+1\right)=-3+\left(-\left(-\frac{1}{2}\right)+5\right)$
$-4x+2+x-1=-3-x+5$	remove parenthesis (distribute)	$2+2-\left(\frac{1}{2}+1\right)=-3+\left(\frac{1}{2}+5\right)$
$-3x+1=-x+2$	simplify	$4-\left(1\frac{1}{2}\right)=-3+\left(5\frac{1}{2}\right)$
$-2x=1$	add x, subtract 1 from each side	$2\frac{1}{2}=2\frac{1}{2}$
$x=-\frac{1}{2}$	divide	

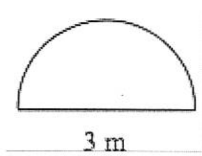
Area of Circles and Complex Figures

For circles, the formulas for area and circumference (perimeter) are:

$A = r^2\pi$ and $C = 2r\pi$ where r = radius of the circle

For complex figures (made of from circles or parts of circles with other shapes), divide the figure into more recognizable parts. Then find the sum of the area of the parts. When finding the perimeter of a complex region, be sure that the sum only includes the edges on the outside of the region.

Example 1:



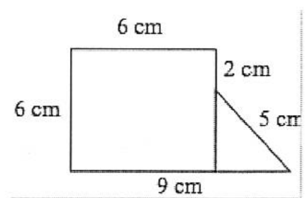
Area of half of a circle:

$A = \frac{1}{2} r^2\pi = \frac{1}{2} \cdot 1.5^2 \cdot \pi \approx 3.53 \text{ m}^2$

Circumference of half of a circle plus the diameter:

$P = \frac{1}{2} \cdot 2r\pi + d = \frac{1}{2} \cdot 2 \cdot 1.5 \cdot \pi + 3 \approx 7.71 \text{ m}$

Example 2:



Area of square plus triangle:

$A = s^2 + \frac{1}{2} bh = 6^2 + \frac{1}{2} \cdot 3 \cdot 4 = 42 \text{ cm}^2$

Add all sides for perimeter:

$6 + 6 + 2 + 5 + 9 = 28$ cm

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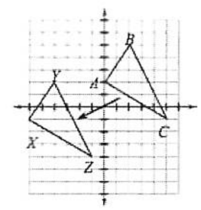
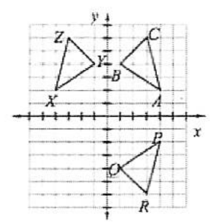
Transformations

Rigid transformations are ways to move an object while not changing its shape or size. A translation (slide) moves an object vertically, horizontally or both. A reflection (flip) moves an object across a line of reflection as in a mirror. A rotation (turn) moves an object clockwise or counterclockwise around a point.
A dilation is a non-rigid transformation. It produces a figure that is similar to the original by proportionally shrinking or stretching the figure from a point.

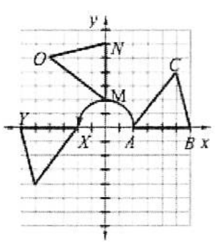
Example 1: Translate (slide) ΔABC left six units and down three units. Give the coordinates of the new ΔXYZ.
Solution: The original vertices are A (0, 2), B (2, 5), and C (5, –1). The new vertices are X (–6, –1), Y (–4, 2), and Z (–1, –4).



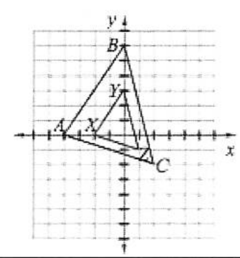
Example 2: Reflect (flip) ΔABC across the x-axis to get ΔPQR . Give the coordinates of the new ΔPQR.
Now reflect (flip) ΔABC across the y-axis to get ΔXYZ. Give the coordinates of the new ΔXYZ.
Solution: The key is that the reflection is the same distance from the axis as the original figure. For the first reflection the points are P $(4, -2)$, Q $(1, -4)$, and R $(3, -6)$. For the second reflection the points are X $(-4, 2)$, Y $(-1, 4)$, and Z $(-3, 6)$.



Example 3: Rotate (turn) ΔABC counterclockwise 90° about the origin to get ΔMNO. Give the coordinates of the new ΔMNO. Then rotate ΔMNO counterclockwise another 90° to get ΔXYZ. Give the coordinates of the new ΔXYZ.
Solution: After the first 90° rotation, the coordinates of A (2,0), B (6,0), and C (5, 4) became M (0, 2), N (0, 6) , and O (–4, 5). Note that each original point (x, y) became $(-y, x)$. After the next 90° rotation, the coordinates of the vertices are now X (–2, 0), Y (–6, 0), and Z (–5, –4). After the 180° rotation each of the points (x, y) in the original ΔABC became $(-x, -y)$. Similarly a 270° counterclockwise rotation or a 90° clockwise rotation about the origin takes each point (x, y) to the point $(y, -x)$.



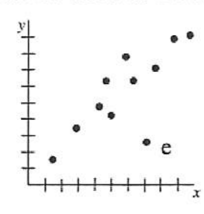
Example 4: Dilate (enlarge/reduce) ΔABC by a scale factor of $\frac{1}{2}$ from the origin to get ΔXYZ. Give the coordinates of the new ΔXYZ.
Solution: Multiplying the coordinates of the vertices of ΔABC by the scale factor gives the coordinates of the vertices of the new ΔXYZ. They are X $(-2, 0)$, Y $(0, 3)$ and Z (1, –1).



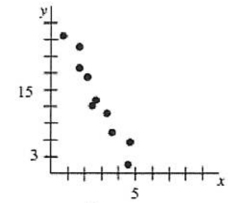
Scatterplots

An association (relationship) between two numerical variables on a graph can be described by its form, direction, strength, and outliers. When the association has a linear form, a line a best fit can be drawn and its equation can be used to make predictions about other data.

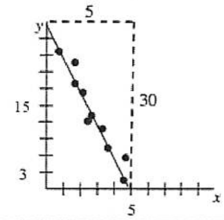
Example 1: Describe the association in the scatterplot at right.
Solution: Looking from left to right, except for point (e), the points are fairly linear and increasing. This is a moderate, positive linear association. The point (e) is an outlier



Example 2: For the scatterplot, draw a line of best fit and determine the equation of the line.
Solution: Use a ruler or straightedge to draw the line that approximates the trend of the points. If it is not a perfect line, approximately the same number of points will be above and below the line of best fit.

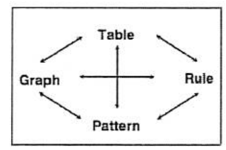


To determine the equation of the line, draw in a slope triangle and determine the ratio of the vertical side to the horizontal side. In this case it is $\frac{-30}{5} = -6$. Estimate the y-intercept by looking at where the line intersects the y-axis. In this case, it is approximately 30. The equation of any non-vertical line may be written in the form $y = mx + b$ where m is the slope and b is the y-intercept.



Multiple Representations of Linear Equations

If you know one representation of a linear pattern, it can also be represented in three other different ways. Linear representations may be written as equations in the form $y = mx + b$ where m is the growth factor and b is the starting value.



Example 1: Using the pattern below, create an $x \rightarrow y$ table, a graph, and write the rule as an equation.

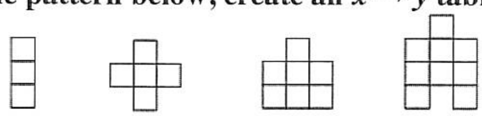
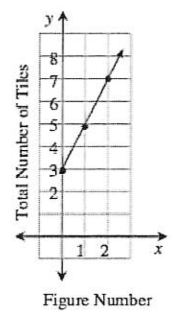


Figure 0 Figure 1 Figure 2 Figure 3

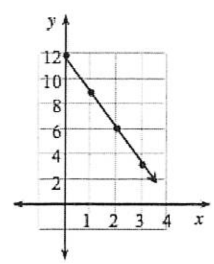
Solution: The number of tiles matched with the figure number gives the $x \rightarrow y$ table. Plotting those points gives the graph. Using the starting value for b and the growth factor for m gives the information to write the equation in $y = mx + b$ form.

Figure # (x)	0	1	2	3
# of Tiles (y)	3	5	7	9



The starting number is 3 tiles and the pattern grows by 2 tiles each figure so the equation is $y = 2x + 3$.

Example 2: Create an $x \rightarrow y$ table and the rule (equation) based on the graph at right.



Solution:

Place the given points in a table:

x	0	1	2	3
y	12	9	6	3

The starting value is 12 and the "growth" factor is -3. The equation is $y = -3x + 12$

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1. Simplify without a calculator:

   a. $6 + 3 \cdot (7 - 3 \cdot 2)(7 - 3 \cdot 2)$

   b. $\frac{14\,(2+3-2\cdot 2)}{4\cdot 4-3\cdot 3}$

2. What did Mikayla do wrong when she solved this equation? What should she have done?

$$\begin{aligned}
-2x + 6 &= 8x + (-4) \\
-4 \quad & \quad -4 \\
-2x + 2 &= 8x \\
+2x \quad & \quad +2x \\
2 &= 10x \\
\tfrac{1}{5} &= x
\end{aligned}$$

3. Evaluate each of the following expressions.

   a. $17 + 3 \cdot 2$

   b. $7 \cdot 2 + 4 - 2(7 + 3)$

   c. $3^2 + 5(1 - 3) + 4 \cdot 5 + 1$

4. Freedom says that $16 - 5 + 3 = 14$ but Rage says $16 - 5 + 3 = 6$. Who is correct? Why?

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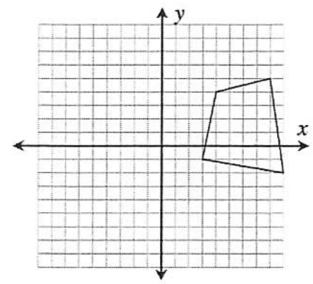
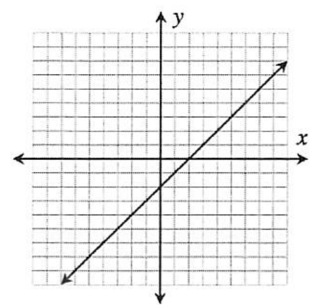
5. Translate the shape 10 units to the left and down 5 units. What are the coordinates for the vertices of the new shape?



6. Examine the graph of the line. Create an equation that when graphed, gives you that line. How do you know your equation is correct?



7. Decide whether each of the following points is on the line $y = -\frac{1}{2}x + 1$. For each point, show your work or explain how you decided.

a. (0, 1) b. (1, 0) c. (– 40, 21) d. (20, –11)

8. If you know the diameter of a circle, how do you find the area of that circle? Explain the steps clearly.

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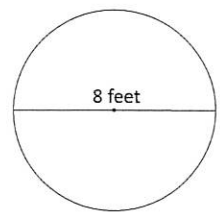
9. What is the diameter of a circle if the circumference is approximately 47.12 feet?

10. Consider the circle shown.



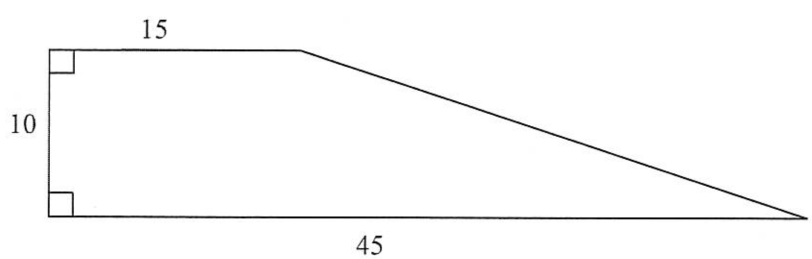
a. What is the length of the radius?

b. What is the length of the diameter?

c. What is the circumference?

d. What is the area?

11. Calculate the area of the shape below. Do you see rectangles or other shapes within this shape? If so, what? Does that help you? Explain.



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12. Use the order of operations to simplify.

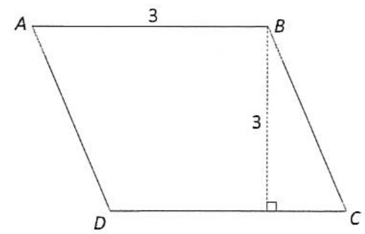
a. $7 \cdot 3 + 8 \cdot 2$

b. $2(14 - 9) - (17 - 14)$

c. $16(2 - 1) \div 2 + 2$

d. $\frac{13+12}{9-4} + 2$

13. In the parallelogram at right,



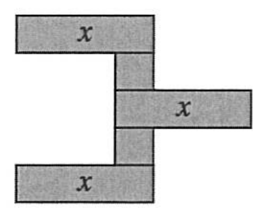
a. Identify the base and the height.

b. What is the area?

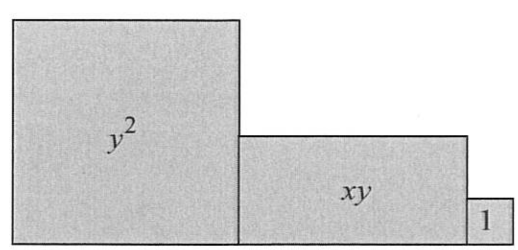
c. Draw a rectangle with the same area as the parallelogram. How do you know it has the same area?

d. Draw a triangle with the same area as the parallelogram. How do you know it has the same area? Can you think of a different one? Come up with at least one other triangle with the same area.

14. James thinks the perimeter of this figure is $6x + 10$. Ana thinks it is $6x + 6$. Who is correct? Explain how you know.



15. Find the perimeter of the shape below if $x = 3$ and $y = 10$.



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16. Evaluate the expression $\frac{16-2^3}{2+2(5-2)}$.

17. Tina can walk 7 blocks in 34 minutes and 23 seconds. About how many blocks can she walk in 2 hours?

18. Solve each of the following equations. Show your work.

a. $2x+3=-x+8$

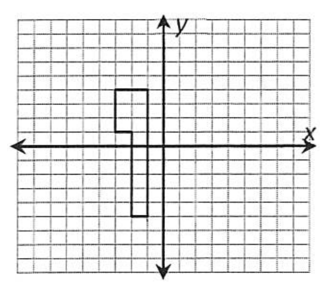
b. $\frac{1}{2}x-3=-7$

c. $\frac{8}{11}=\frac{3}{2y}$

d. $\frac{x+5}{3}=\frac{x+5}{7}$

e. $8=-4+3(3x+5)$

19. A polygon is shown on the coordinate grid. Draw the result of this polygon after translating it right five units, and down two units. What will be the new coordinate of the vertex at (–3, 4) after translation?



20. Simplify the following expressions and decide which expression has the greatest value.

a. $(-6)^2+3^2$

b. $(-36 \div 6)+(42 \div -7)$

c. $(-9)(-9)(8)(-2)$

d. $-(9)-(-9)+6+(6)$

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21. Solve for x: $\frac{2}{5}x+\frac{1}{3}=1$.

22. Solve for x: $\frac{3}{5}x+\frac{1}{3}=1$

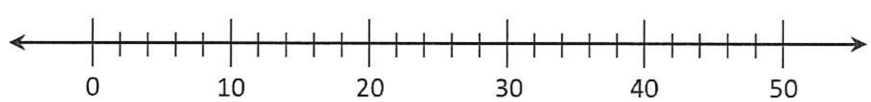
23. Solve for x.

a. $\frac{2}{7}x=4$

b. $\frac{1}{3}x+2=4$

24. On the number line below, make a box-and-whisker plot for this data.

2, 7, 9, 12, 14, 22, 32, 36, 43



a. What is the median for this data? Label it on the graph.

b. What is the lower quartile? Label it on the graph.

c. What is the upper quartile? Label it on the graph.

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25. Bamboo is a fast growing plant. One type, Black Bamboo, grows at a rate of 11 cm per month, while another, Golden Bamboo, grows at a rate of 1.2 meters per year. Which bamboo grows faster? Why?

26. Farmer Janet can plant nine feet of carrots in 15 minutes while her daughter Amy can plant 17 feet of carrots in half an hour.

    a. Which farmer plants carrots more quickly? Why?

    b. What is each farmer's rate in carrots per hour?

27. Find the rates for each of the following situations. Show your work on each.

    a. Jennifer babysat for 7 hours and earned $52.50. What is her hourly rate of pay?

    b. Zeke drove 346 miles in 8 hours. What is his average rate of speed (mph)?

    c. Will bought 14 feet of rope for $ 10.50. What does the rope cost, per foot?

    d. What math operation do all of the above problems have in common?

28. Noelle and Tanya like to make latkes (potato pancakes) for their friends. They can make 12 latkes in 10 minutes.

    a. At that rate, how long will it take them to make 200 latkes? Be sure to organize, label, and justify your work.

    b. Now reverse the problem. How many latkes can they make in an hour? Show all work.

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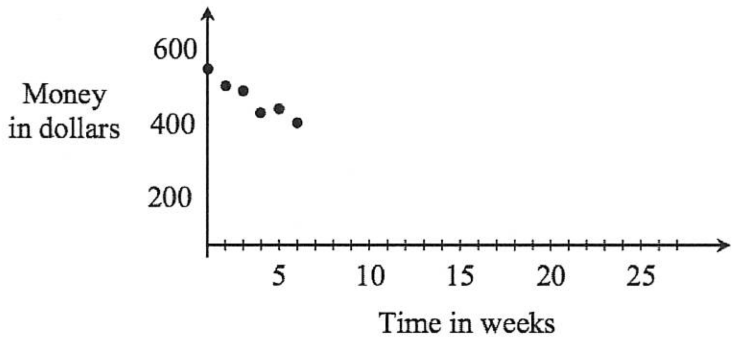
29. The graph below shows the math club's money during the first 6 weeks of school. At this rate, when will the club have no money left? Justify your answer.



30. Examine this tile pattern and answer the questions below.

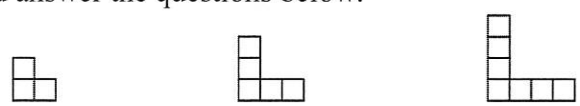
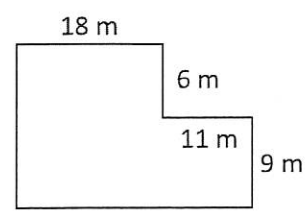


Figure 1 Figure 2 Figure 3

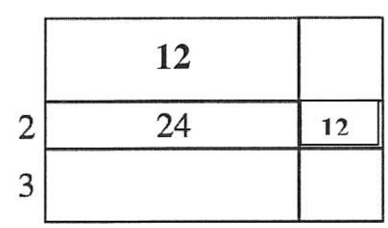
a. Write a rule that describes this pattern.

b. Which figure number has 41 squares? Show or explain how you figured it out.

31. Each side of the figure is either horizontal or vertical. Calculate the area and the perimeter. Show any rectangles you see and use in your solution.



32. Find the perimeter and area of the entire rectangle at right. Note that the areas of some of the parts have been labeled inside the rectangle.



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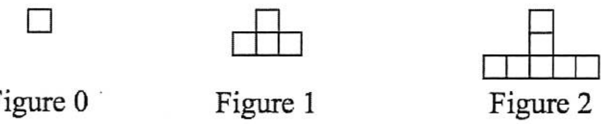
33. Study the tile pattern below.



a. Draw Figure 3 and Figure 4. Explain how the pattern grows.

b. Write an equation (rule) for the number of tiles in the pattern.

c. Explain how the growth factor appears in your equation.

34. Create the first three figures (Figure 0, Figure 1, and Figure 2) of a tile pattern that follows the rule $y = 2x + 3$. Draw each figure so that the 2, x and 3 are easy to see.

35. The freshman class at Christian Brothers High School is having an election. Cara randomly asked 50 students whom they would vote for, and 30 said that Mckenzie was their choice. If all 350 members of the freshman class vote, how many votes can you expect Mckenzie to receive? Show how using a table like the one at the right can help you answer the question. Show all your work.

# of Potential Voters	# of Votes Expected for Mckenzie
50	30
350	?

36. Solve each equation below.

a. $2.5 = \frac{x}{5}$ b. $\frac{4}{9} = \frac{x}{10}$ c. $\frac{x}{1} = \frac{2x+1}{100}$ d. $\frac{5-x}{9} = \frac{2}{3}$

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37. Graph the following rules on one set of axes. Label each line with its equation, the y-intercept, and a growth triangle.

a. $y = 4x - 3$ b. $y = -2x + 5$ c. $y = -5x - 1$

38. Create a table, graph, and equation to match the tile pattern below. Use one of your representations to determine the number of tiles in the 100th figure.

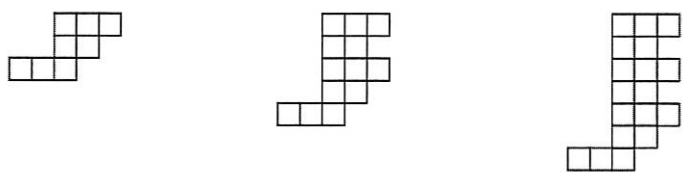

Figure 2 Figure 3 Figure 4