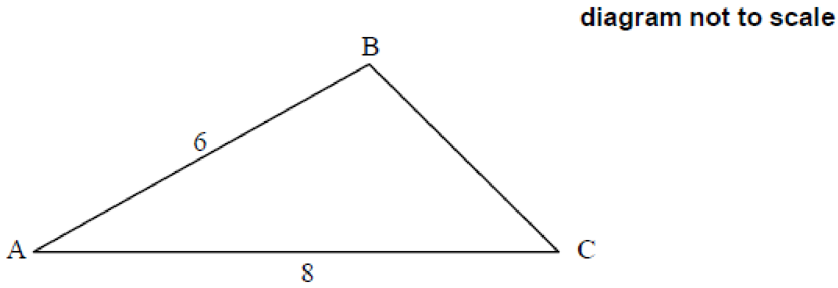
Analysis 12 HL Summer Review 2020 *[110 marks]*

The following diagram shows triangle ABC, with $AB = 6$ and $AC = 8$.



1a. Given that $\cos \hat{A} = \frac{5}{6}$ find the value of $\sin \hat{A}$. *[3 marks]*

1b. Find the area of triangle ABC. *[2 marks]*

2a. Show that $\log_9(\cos 2x + 2) = \log_3\sqrt{\cos 2x + 2}$. *[3 marks]*

2b. Hence or otherwise solve $\log_3(2\sin x) = \log_9(\cos 2x + 2)$ for $0 < x < \frac{\pi}{2}$. *[5 marks]*

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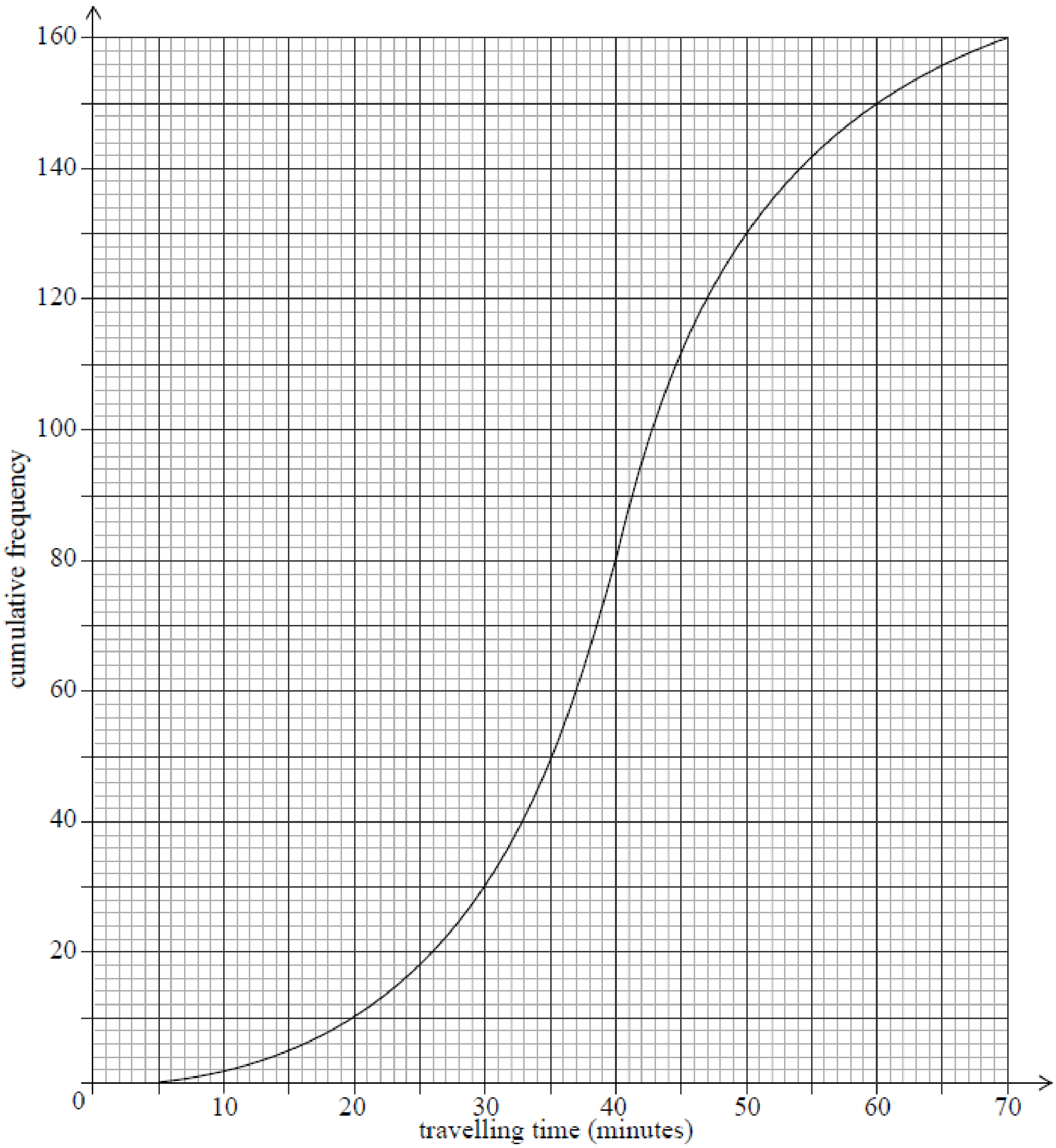
A large company surveyed 160 of its employees to find out how much time they spend traveling to work on a given day. The results of the survey are shown in the following cumulative frequency diagram.



3a. Find the median number of minutes spent traveling to work. *[2 marks]*

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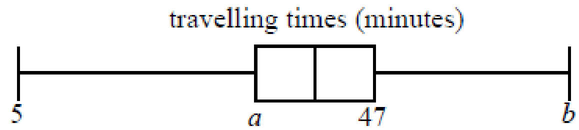
3b. Find the number of employees whose travelling time is within 15 minutes of the median. *[3 marks]*

Only 10% of the employees spent more than k minutes traveling to work.

3c. Find the value of k. *[3 marks]*

The results of the survey can also be displayed on the following box-and-whisker diagram.



3d. Write down the value of b. *[1 mark]*

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3e. Find the value of a. *[2 marks]*

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3f. Hence, find the interquartile range. *[2 marks]*

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3g. Travelling times of less than p minutes are considered outliers. *[2 marks]*

Find the value of p.

In an arithmetic sequence, $u_2 = 5$ and $u_3 = 11$.

4a. Find the common difference. *[2 marks]*

4b. Find the first term. *[2 marks]*

4c. Find the sum of the first 20 terms. *[2 marks]*

Let $g\left(x\right) = x^2 + bx + 11$. The point $\left(-1,8\right)$ lies on the graph of g.

5a. Find the value of b. *[3 marks]*

5b. The graph of $f(x) = x^2$ is transformed to obtain the graph of g. *[4 marks]*

Describe this transformation.

Consider $\begin{pmatrix} 11 \\ a \end{pmatrix} = \frac{11!}{a!\,9!}$.

6a. Find the value of a. *[2 marks]*

6b. Hence or otherwise find the coefficient of the term in x^9 in the expansion of $(x+3)^{11}$. *[4 marks]*

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The points A and B have position vectors $\begin{pmatrix} -2 \\ 4 \\ -4 \end{pmatrix}$ and $\begin{pmatrix} 6 \\ 8 \\ 0 \end{pmatrix}$ respectively.

Point C has position vector $\begin{pmatrix} -1 \\ k \\ 0 \end{pmatrix}$. Let O be the origin.

Find, in terms of k,

7a. $\overrightarrow{\mathrm{OA}} \bullet \overrightarrow{\mathrm{OC}}$. *[2 marks]*

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7b. $\overrightarrow{\mathrm{OB}} \bullet \overrightarrow{\mathrm{OC}}$. *[1 mark]*

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7c. Given that $A\widehat{O}C = B\widehat{O}C$, show that $k = 7$. *[8 marks]*

7d. Calculate the area of triangle AOC. *[6 marks]*

A line, L_1, has equation $r = \begin{pmatrix} -3 \\ 9 \\ 10 \end{pmatrix} + s \begin{pmatrix} 6 \\ 0 \\ 2 \end{pmatrix}$. Point $\mathrm{P}\,(15,\ 9,\ c)$ lies on L_1.

8a. Find c. *[4 marks]*

8b. A second line, L_2, is parallel to L_1 and passes through (1, 2, 3). *[2 marks]*
Write down a vector equation for L_2.

The lengths of two of the sides in a triangle are $4\,\text{cm}$ and $5\,\text{cm}$. Let θ be the angle between the two given sides. The triangle has an area of $\frac{5\sqrt{15}}{2}$ cm^2.

9a. Show that $\sin\theta = \frac{\sqrt{15}}{4}$. *[1 mark]*

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9b. Find the two possible values for the length of the third side. *[6 marks]*

10. Solve the simultaneous equations *[7 marks]*

$$\log_2 6x = 1 + 2\log_2 y$$

$$1 + \log_6 x = \log_6 (15y - 25).$$

A group of 7 adult men wanted to see if there was a relationship between their Body Mass Index (BMI) and their waist size. Their waist sizes, in centimetres, were recorded and their BMI calculated. The following table shows the results.

Waist (x **cm**)	58	63	75	82	93	98	105
BMI (y)	19	20	22	23	25	24	26

The relationship between x and y can be modelled by the regression equation $y = ax + b$.

11a. Write down the value of a and of b. *[3 marks]*

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11b. Find the correlation coefficient. *[1 mark]*

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11c. Use the regression equation to estimate the BMI of an adult man whose waist size is 95 cm. *[2 marks]*

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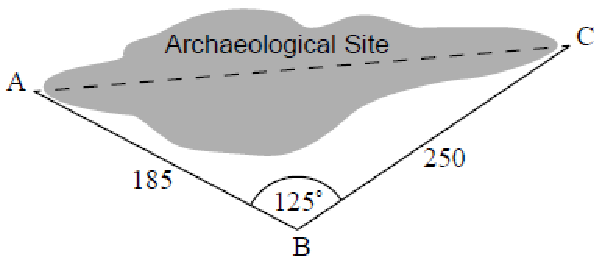
An archaeological site is to be made accessible for viewing by the public. To do this, archaeologists built two straight paths from point A to point B and from point B to point C as shown in the following diagram. The length of path AB is 185 m, the length of path BC is 250 m, and angle $\mathrm{A}\hat{\mathrm{B}}\mathrm{C}$ is 125°.

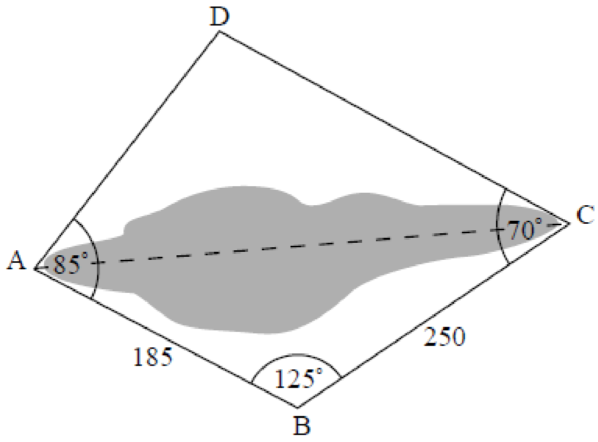
diagram not to scale



12a. Find the distance from A to C. *[3 marks]*

The archaeologists plan to build two more straight paths, AD and DC. For the paths to go around the site, angle $\mathrm{B}\hat{\mathrm{A}}\mathrm{D}$ is to be made equal to 85° and angle $\mathrm{B}\hat{\mathrm{C}}\mathrm{D}$ is to be made equal to 70° as shown in the following diagram.

diagram not to scale



12b. Find the size of angle $\mathrm{B}\hat{\mathrm{A}}\mathrm{C}$. *[3 marks]*

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12c. Find the size of angle $\mathrm{C}\hat{\mathrm{A}}\mathrm{D}$. *[1 mark]*

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12d. Find the size of angle $\mathrm{A}\hat{\mathrm{C}}\mathrm{D}$. *[2 marks]*

12e. The length of path AD is 287 m. *[4 marks]*

Find the area of the region ABCD.

13a. Find the roots of the equation $w^3 = 8\mathrm{i}$, $w \in \mathbb{C}$. Give your answers in Cartesian form. *[4 marks]*

13b. One of the roots w_1 satisfies the condition $\mathrm{Re}\,(w_1) = 0$. *[3 marks]*

Given that $w_1 = \frac{z}{z-\mathrm{i}}$, express z in the form $a + b\mathrm{i}$, where a, $b \in \mathbb{Q}$.